Attachment #2

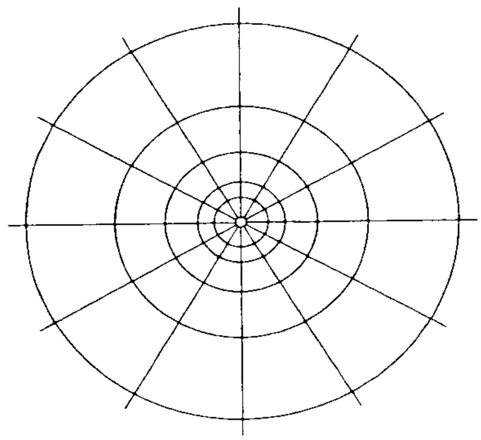

Fig. 7.26 Flow net for source or vortex.

Lines of constant ϕ (equipotential lines) and constant ψ are shown in Fig. 7.26. A *sink* is a negative source, a line into which fluid is flowing.

vortex

In examining the flow case given by selecting the stream function for the source as a velocity potential,

$$\phi = -\mu\theta \qquad \psi = \mu \ln r$$

which also satisfies the Laplace equation, it is seen that the equipotential lines are radial lines and the streamlines are circles. The velocity is in a tangential direction only, since $\partial\phi/\partial r = 0$. It is

$$q = -\left(\frac{1}{r}\right)\frac{\partial\phi}{\partial\theta} = \frac{\mu}{r}$$

since $r\,\delta\theta$ is the length element in the tangential direction.

In referring to Fig. 7.27, the *flow along a closed curve* is called the *circulation*. The flow along an element of the curve is defined as the product of the length element δs of the curve and the component of the velocity tangent to the curve, $q \cos \alpha$. Hence the circulation Γ around a closed path C is

$$\Gamma = \int_C q \cos \alpha \, ds = \int_C \mathbf{q} \cdot d\mathbf{s}$$

The velocity distribution given by the equation $\phi = -\mu\theta$ is for the *vortex*

Attachment #2 cont'd

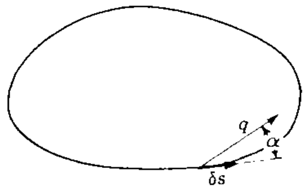

Fig. 7.27 Notation for definition of circulation.

and is such that the circulation around any closed path that contains the vortex is constant. The value of the circulation is the strength of the vortex. By selecting any circular path of radius r to determine the circulation, $\alpha = 0°$, $q = \mu/r$, and $ds = r\,d\theta$; hence,

$$\Gamma = \int_C q \cos \alpha \, ds = \int_0^{2\pi} \frac{\mu}{r} r \, d\theta = 2\pi\mu$$

At the point $r = 0$, $q = \mu/r$ goes to infinity; hence, this point is called a singular point. Figure 7.26 shows the equipotential lines and streamlines for the vortex.

doublet

The two-dimensional doublet is defined as the limiting case as a source and sink of equal strength approach each other so that the product of their strength and the distance between them remains a constant $2\pi\mu$. μ is called the *strength* of the doublet. The axis of the doublet is from the sink toward the source, i.e., the line along which they approach each other.

In Fig. 7.28 a source is located at $(a,0)$ and a sink of equal strength at

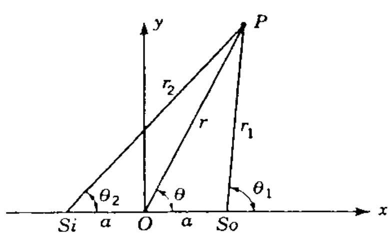

Fig. 7.28 Notation for derivation of two-dimensional doublet.